B.A./B.Sc. 5th Semester (General) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMG5DSE1A1

(Matrices)

Time : 3 Hours **Full Marks : 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meanings.

1. Answer *any ten* questions: 2×10=20

(a) Define rank of a matrix.

(b) Define linear independence of a set of vectors in a vector space.

(c) Prove that the subset T defined by $T = \{(x, y, z) \in \mathbb{R}^3 : x = 1\}$ is not a subspace of $\mathbb{R}^3$.

(d) Prove that the set of vectors $\{(1,2,2), (2,1,2), (2,2,1)\}$ is linearly independent in $\mathbb{R}^3$.

(e) Define basis in a vector space.

(f) Express (–1, 2, 4) as a linear combination of (–1, 2, 0), (0, –1, 1), (3, –4, 2) in the vector space $\mathbb{R}^3$.

(g) Prove that a subset of a linearly independent set of vectors in a vector space is linearly independent.

(h) Define elementary operations on a given matrix.

(i) Define eigenvalues and eigenvectors of a matrix.

(j) Prove that any set of vectors containing the null vector is linearly dependent in a vector space.

(k) Define eigenspace corresponding to an eigenvalue of a matrix.

(l) State the conditions for the existence of solutions of a system of non-homogeneous equations.

(m) Define Dilation and give an example of it.

(n) Define normal form of a matrix.

(o) Write down the standard basis of the vector space $\mathbb{R}^n$ over $\mathbb{R}$.

2. Answer *any four* questions: 5×4=20

(a) Find a basis for the vector space $\mathbb{R}^3$ that contains the vectors (1, 2, 0), (1, 3, 1). 5

(b) Find a basis and dimension of the subspace W of $\mathbb{R}^3$, where
$W = \{(x, y, z): x + 2y + z = 0, 2x + y + 3z = 0\}$. 5

(c) Determine the rank of the following matrix by using elementary operations: 5

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{pmatrix}$$

(d) Find the inverse of the following matrix by using elementary row operations: 5

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{pmatrix}$$

(e) Let S be the set defined by $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$. Prove that S is not a subspace of $\mathbb{R}^3$. 5

(f) Find k so that the set of vectors $\{(1, -1, 2), (0, k, 3), (-1, 2, 3)\}$ is linearly dependent in $\mathbb{R}^3$. 5

3. Answer *any two* questions: 10×2=20

(a) (i) Obtain the fully reduced normal form of the following matrix:

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

(ii) Prove that a matrix A is non-singular if and only if A is row equivalent to the identity matrix. 7+3

(b) (i) Solve the system of following equations:

$$x + 2y + z = 1$$

$$3x + y + 2z = 3$$

$$x + 7y + 2z = 1$$

(ii) Prove that a necessary and sufficient condition for a non-homogeneous system $AX=B$ to be consistent is that rank of A= rank of $\bar{A}$, where $\bar{A}$ is the augmented matrix. 6+4

(c) (i) Prove that the rank of a matrix remains invariant under an elementary row operation.

(ii) If the set of vectors $\{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\}$ in a vector space is linearly dependent, then prove that at least one of the vectors of the set can be expressed as a linear combination of the remaining others and conversely. 5+5

(d) (i) Define rotation and translation.

(ii) Find the matrix A that rotates every vector through an angle 45 degree in $\mathbb{R}^2$. Also find the eigenvalues and eigenvectors of this matrix. 3+3+(2+2)

B.A./B.Sc. 5th Semester (General) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMG5DSE1A2

(Mechanics)

Time : 3 Hours **Full Marks : 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) If the radial velocity is proportional to the transverse velocity, find the path in polar coordinates.

(b) Define limiting friction.

(c) Define centre of gravity of a system of particles.

(d) State the principle of linear momentum for a system of particles.

(e) Distinguish between conservative and non-conservative forces.

(f) Show that the work done by the force is equal to the product of the impulse and the mean of the initial and final velocities.

(g) Derive the expression for potential energy of a simple pendulum of length l oscillating in a uniform gravitational field.

(h) Find the resultant of two simple harmonic motions having slightly different periods.

(i) Define impulsive force. How is it measured?

(j) The maximum height attained by a projectile is equal to its range. Find the direction of projection.

(k) A particle describes the curve $r = ae^{\theta}$ with constant angular velocity. Show that the radial acceleration is zero.

(l) State the principle of conservation of energy.

(m) What is the work done by gravity on a stone of mass 80 gm during the 8th second of its fall?

(n) If a man can throw a ball h meters vertically upwards, show that the greatest horizontal distance he can throw it is $2h$.

(o) State Newton's second law of motion.

2. Answer *any four* questions: 5×4=20

(a) A particle describes a circle of radius a with a uniform speed v; show that at any instant its acceleration is directed towards the centre and is of magnitude $\frac{v^2}{a}$. 5

(b) A rough wire which has the shape of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is placed with its x-axis vertical and y-axis horizontal. If μ be the co-efficient of friction, find the depth below the highest point of the position of limiting equilibrium of a bead which rest on the wire. 5

(c) A particle is projected with velocity u at an inclination α above the horizontal in a medium whose resistance per unit mass is k times the velocity. Show that its direction will again make an angle α below the horizontal after a time $\frac{1}{k}\log(1+\frac{2ku}{g}\sin\alpha)$. 5

(d) Find the radial and transverse components of acceleration of a particle moving along a plane curve. 5

(e) Show that in a simple harmonic motion of amplitude a and period T and the velocity v at a distance x from the centre is given by $v^2T^2=4\pi^2(a^2-x^2)$. Find the new amplitude if the velocity were doubled when the particle is at a distance $\frac{a}{2}$ from the centre, the period remaining unaltered. 3+2

(f) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to height h. Show that the velocity of recoil of the gun is $\left\{\frac{2m^2gh}{M(m+M)}\right\}^{\frac{1}{2}}$. 5

3. Answer *any two* questions: 10×2=20

(a) (i) Three forces P, Q, R act along the sides of a triangle formed by the lines $x+y=3$, $2x+y=1$ and $x-y+1=0$. Find the equation of the line of action of the resultant.

(ii) Find the centre of gravity of a segment of a circular disc subtend an angle 2α at the centre. 5+5

(b) (i) A hemispherical shell is on a rough plane, whose angle of friction is λ. Show that the inclination of the plane base of the rim to the horizontal cannot be greater than $\sin^{-1}(2\sin\lambda)$.

(ii) Find the tangential and normal components of acceleration of a particle moving along a plane curve. 5+5

(c) (i) A point P describes, with constant angular velocity an equiangular spiral of which O is the pole. Find its acceleration and show that its direction makes the same angle with the tangent at P as the radius vector OP makes with the tangent.

(ii) A particle executing simple harmonic motion in a straight line has velocities v_1, v_2, v_3 respectively at distances x_1, x_2, x_3 from the centre of the path. Prove that $x_1^2(v_2^2 - v_3^2) + x_2^2(v_3^2 - v_1^2) + x_3^2(v_1^2 - v_2^2) = 0$. 5+5

(d) (i) A body of mas $(m_1 + m_2)$ is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E; show that if after explosion the parts move in the same line as before, their relative speed is $\left\{\frac{2E(m_1+m_2)}{m_1 m_2}\right\}^{\frac{1}{2}}$.

(ii) Deduce the expressions for the tangential and normal components of the velocities and accelerations of a particle moving on a plane curve. 5+5

B.A./B.Sc. 5th Semester (General) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMG5DSE1A3

(Linear Algebra)

Time : 3 Hours **Full Marks : 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meanings.

1. Answer *any ten* questions: 2×10=20

(a) Define basis of a vector space.

(b) Find the eigenvalues of the matrix $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$.

(c) When is a linear transformation said to be an isomorphism?

(d) Show that T is non-singular, where $T(x,y,z) = (x-y, x+2y, y+3z), (x,y,z) \in \mathbb{R}^3$.

(e) Show that eigenvalues of a diagonal matrix are its diagonal elements.

(f) In $\mathbb{R}^2$, $\alpha = (3,1)$, $\beta = (2,-1)$. Determine linear span of α, β.

(g) Show that $T: \mathbb{R}^3 \to \mathbb{R}^3$ is not a linear mapping where
$T(x_1,x_2,x_3) = (x_1+1, x_2+1, x_3+1), (x_1,x_2,x_3) \in \mathbb{R}^3$.

(h) Find Ker T and Im T of a linear mapping T, where $T(x,y) = (x+y, x-y)$, $(x,y) \in \mathbb{R}^2$.

(i) If a linear transformation $T: V \to W$ be invertible, then prove that T has a unique inverse.

(j) If T is one-to-one, then prove that T is onto, where $T: V \to V$ is a linear mapping and V is a finite dimensional vector space.

(k) Let $E = \{(1,0),(0,1)\}$ and $S = \{(1,3),(1,4)\}$. Find the change of basis matrix from E to S.

(l) Show that the eigenvalues of a real skew symmetric matrix are purely imaginary.

(m) What do you mean by double dual of a vector space?

(n) Find T^{-1}, where $T: \mathbb{R} \to \mathbb{R}$ be defined by $T(x) = 2x - 3$.

(o) Find the characteristic polynomial of $T(x,y,z) = (2x+y-2z, 2x+3y-4z, x+y+z)$, where $T: \mathbb{R}^3 \to \mathbb{R}^3$ is a linear operator.

2. Answer *any four* questions: 5×4=20

(a) Let $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$. Find all eigenvalues of A and corresponding eigenvectors. 1+4

(b) Find the dual basis of the basis $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ of $\mathbb{R}^3$.

(c) Let V and W be two finite dimensional vector spaces over a field F. Then prove that V and W are isomorphic if and only if $\dim V = \dim W$.

(d) Find a basis and the dimension of Kernel of G and the dimension of the image of G, where $G: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $G(x, y, z) = (x + y + z, 2x + 2y + 2z)$.

(e) (i) Let V be the real vector space of functions $f: \mathbb{R} \to \mathbb{R}$ and let $W = \{f: \mathbb{R} \to \mathbb{R}$ such that $f(-x) = -f(x)\}$. Then show that W is a subspace of V.

(ii) Show that the vectors $u_1 = (1, 1, 1), u_2(1, 2, 3), u_3(1, 5, 8)$ span $\mathbb{R}^3$. 2+3

(f) (i) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$, be a linear transformation such that $T(1, 1) = (2, -3)$ and $T(1, -1) = (4, 7)$. Find the matrix of T relative to the basis $\{(1, 0), (0, 1)\}$.

(ii) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$. Find the matrix representation of T relative to the basis $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $\{(1, 3), (2, 5)\}$ 2+3

3. Answer *any two* questions: 10×2=20

(a) (i) Let V be a vector space of dimension n over a field $\mathbb{R}$. Then prove that V is isomorphic to $\mathbb{R}^n$.

(ii) Let S and T be linear mappings of $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

$S(x, y, z) = (z, y, x)$ and $T(x, y, z) = (x + y + z, y + z, z), (x, y, z) \in \mathbb{R}^3$. Find TS and ST.

5+5

(b) (i) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x_1, x_2, x_3) = (2x_1 + x_2 - x_3, x_2 + 4x_3, x_1 - x_2 + 3x_3)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$. Find the matrix of T relative to the order basis $\{(1,0,0), (0,1,0), (0,0,1)\}$ of $\mathbb{R}^3$.

(ii) If λ be an eigenvalue of an $n \times n$ idempotent matrix P, then prove that λ is either 1 or 0.

5+5

(c) (i) Determine the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$ to (1, 1, 1), (1, 1, 1), (1, 1, 1) respectively. Verify that $\dim(\text{Ker } T) + \dim(\text{Im } T) = 3$.

(ii) Prove that a linear mapping $T: V \to W$ is invertible if and only if T is one-to-one and onto. 5+5

(d) (i) Let V be the set of all $m \times n$ matrixes over $\mathbb{R}$. Prove that V is a real vector space.

(ii) Prove that the subspace $U + W$ is the smallest subspace of a vector space V containing the subspaces U and W of V. 5+5
